From (1), $z=.00066$. $s=6416.9M$. The ultimate strength of hard steel is 240000 lb./in.^{2}.

$\therefore s=240000/f=240000/6=40000$ lb./in.^{2}

$\therefore 40000=6416.9M$. $\therefore M=6.2335$.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

159. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

Show that if the equation $y^3=2x^2-1$ be possible in integers, $y=24n^2-1$, or $2n^2-1$, and find three solutions.

Solution by the PROPOSER.

The equation may be written $(y+1)(y^2-y+1)=2x^2$.

Since y^2-y+1 is always odd, it is evident that $y+1$ must be even. Since $y^2-y+1=(y+1)(y-2)+3$ it is evident that $y+1$ and y^2-y+1 can have no common factor but 3. Therefore we have the following possibilities for y: $y=2\times 3m^2-1$, or $y=2n^2-1$.

Since y^3 is represented by the form $2x^2-1$, 2 must be a quadratic residue of y. Therefore $y=8a\pm 1$, and this is possible in the first expression only when $m=2n$. Then either $y=24n^2-1$ or $2n^2-1$.

Substituting these values of y in the original equation, we have

$$192n^4-24n^2+1=r^2 \text{ or } 4n^4-6n^2+3=r^2.$$

The first equation has the solution $n=0$ and 1 which give $y=-1$, $x=0$; $y=23$, $x=78$.

The second equation has the solution $n=1$, which gives $y=1$, $x=1$.

AVERAGE AND PROBABILITY.

198. Proposed by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

Find the average length of a hole at random through a given cylinder.

No solution of this problem has been received.

199. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

A circle is inscribed in a given square. Two points are taken at random within the square but without the circle. What is the chance the distance between the points does not exceed the side of the square?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

This is the same as 196, but as there is a distance less than the side of the square when both points are taken one each in opposite corners, it is